First Order ODE Solutions

Some equations can be solved using multiple methods

Separable $P(x)dy + Q(y)dy = 0$

Steps:

Set equal to each other and solve

Linear $\frac{dy}{dx} + P(x)y = f(x)$

Steps:

Find integrating factor: $\mu = e^{\int P dx}$

$$\frac{d}{dx}[\mu * \text{y}] = f(x) * \mu$$

Integrate both sides

$$\mu * \text{y} = \int (f * \mu) dx$$

Exact $M(x, y)dx + N(x, y)dy = 0$

where $\frac{\partial \text{M}}{\partial \text{y}} = \frac{\partial \text{N}}{\partial \text{x}}$

Steps:

Find $\int M dx$ and $\int N dy$

$$\int M dx + g(y) = \int N dy + h(x)$$

Copy each unique component and set equal to C

Ex: $x^2 + 2 + g(y) = x^2 + y^2 + h(x)$ (Don't copy the second x^2)

=> $x^2 + y + 2 = C$

Homogeneous $M(x,y)dx + N(x,y)dy = 0$

where *M* and *N* are to the same power

Steps:

$y = ux$

$$\frac{dy}{dx} = u\frac{dx}{dx} + x\frac{du}{dx} => dy = udx + xdu$$

or

$x = uy$

$$\frac{dx}{dy} = u\frac{dy}{dy} + y\frac{du}{dy} => dx = udy + ydu$$

Substitute for dy or dx

Becomes separable

Solve as separable equation

Bernoulli $\frac{dy}{dx} + P(x)y = f(x)y^n$

where $n \neq 0 \; or \; 1$

Steps:

$u = y^{1-n}$

Solve for y

$y = u^m$

Differentiate y: $\frac{dy}{dx} = m * u^m \frac{du}{dx}$

Substitute for y and $\frac{dy}{dx}$

Should become linear

Find integrating factor: $\mu = e^{\int Pdx}$

$$\frac{d}{dx}[\mu * \mathrm{u}] = f(x)$$

Integrate both sides

$$\mu * \mathrm{u} = \int f(x)dx$$

Substitute y for u

Reduction to separation of variables $\frac{dy}{dx} = f(Ax + By + C)$

Steps:

Substitute $u = Ax + By + C$

Result is separable

Solve as separable equation

Substitute $Ax + By + C$ for u