Higher Order ODE Models

Undamped motion

$\frac{d^2x}{dt^2} + \omega^2 x = 0$

Where $\omega^2 = k/m$

General solution:

$x(t) = c_1 cos(\omega t) + c_2 sin(\omega t)$

Can be rewritten as $x = Ccos(\omega t - \gamma)$

Where $C = \sqrt{c_1^2 + c_2^2}$ and $\gamma = tan^{-1}(\frac{B}{A})$

Free Damped Motion

$m\frac{d^2x}{dt^2} + \beta\frac{dx}{dt} + kx = 0 \Rightarrow \frac{d^2x}{dt^2} + \frac{\beta}{m}\frac{dx}{dt} + \frac{kx}{m} = 0$

Where β is the damping constant

Driven Motion

$m\frac{d^2x}{dt^2} + \beta\frac{dx}{dt} + kx = f(t)$

Where $f(t)$ is the driving motion

LRC-Series Circuits

$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = E(t)$

$\frac{dq}{dt} = i$